



## Determining mathematical relations among important characteristics of *Fraxinus excelsior* (Case study: Research-Training Forest of Kheyroud)

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### Abstract

In forest management, determining characteristics of trees is essential for assessing tree volume and providing other mathematical equations. In this study, the characteristics of 256 trees including stump diameter, diameter at breast height, total height, crown height, height of trunk with and without branch were measured in an even-aged *Fraxinus* stand at Chelir part of Kheyroud Forest in north of Iran. Results showed there are significant statistical correlations among various characteristics of *Fraxinus excelsior* trees, and mathematic-statistical equations were established among these characteristics. We found the incremental power relation between stump diameter ( $d_{st}$ ) and diameter at breast height ( $d$ ), which can be used for estimating diameter at breast height of cut trees. Also, another equation was established between diameter at breast height and total height ( $h_t$ ) which is used for estimating total height of those trees for which diameter at breast height is available. An incremental logarithmic relation was found between diameter at breast height and crown height ( $h_c$ ) and an inverse one between diameter at breast height and trunk height ( $trh$ ) for this species. Also, a sigmoid relation was found between stump diameter and total height. Results implied that trunk height decreases and crown height increases as total height increases. Trees with higher diameters have high height of trunk without branch. The relationship between Slenderness coefficient ( $h/d$ ) and diameter at breast height of the trees was found to be a power type.

**Keywords:** Even-aged stand, total height, without branch trunk height, power relation, sigmoid relation, inverse relation

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## 1. Introduction

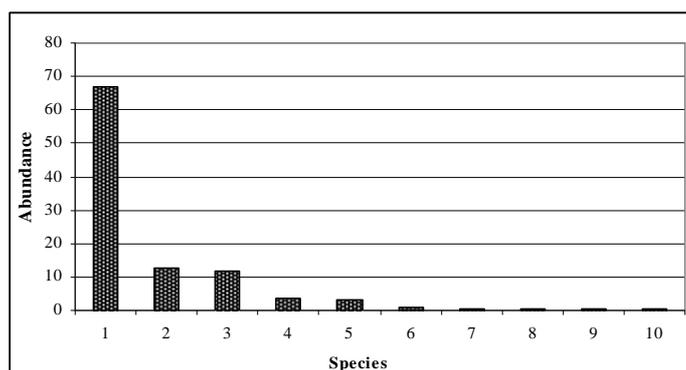
Growth properties of forest trees are defined via various external and internal factors. Physiology, species nature, tree maturity, growth form in its life cycle, genetic characteristics and parent trees' quality and climatic condition, aspect, soil type, absorption of minerals and water uptake, distance among trees, growth form of the adjacent trees and the location of the certain tree among other trees are defined as internal and external elements respectively. The assemblage of these factors forms the growth of certain trees (Marvi-Mohajer, 2005). These factors are of great importance for lateral growth of trees. During recent decades, modern forestry has emerged with the core idea of reaching ecological stability and the trend of developing this scope of forestry science is fast expanding (Spence, 2001). Thus, preserving forest stands could be thought of as the principal objective of the modern forestry (Pommerening, 2002). In order to meet these objectives, there is a need for comprehensive studies of various forest tree species under different circumstances and identify the growth form and the relationship between the sizes of different tree species. Namiranian (2000) carried out a test on 124 trees of *Fagus sylvatica* in Gorazbon Educational Forests in Iran, to establish relationships between different parameters. His results showed an increasing linear relationship between the stump diameter and the diameter at breast height, an increasing parabolic relationship between the diameter at breast and the height of canopy and a significant statistical relationship between the diameter at breast height and canopy diameter.

Further investigation and experimentation by Namiranian in 2004, resulted in a formula as  $d = 0.397079 + 0.773406d_{st}$  between the diameter at breast height ( $d$ ) and the stump diameter ( $d_{st}$ ) of 116 trees of *Fraxinus excelsior*. Akhavan and Namiranian (2007) showed that there was a decreasing exponential relationship between the diameter at breast height and the slenderness coefficient of the trees. Namiranian (2000, 2004) made an effort to examine the slenderness coefficient of *Fagus sylvatica* and *Fraxinus excelsior* at an all-age stand in Gorazbon Educational Forests in Kheyroud, and reached a conclusion that there was a decreasing relationship between the diameter at breast height and the slenderness coefficient for these species. Regarding the fact that the validity of the established formulas is site specific, current work seeks to determine relationships between size dependent characteristics of the forest trees in a pristine condition for *Fraxinus excelsior* in the given region.

## 2. Materials and Methods

The study is conducted in Educational Forests of Kheyroud, located seven kilometers away from Noshahr, Mazandaran Province. This forest area could be divided into seven sections and the required data was gathered in a natural even-aged stand with an area of one hectare in Chelirpart. It is worth noticing that the stand is virtually untouched and there haven't been any construction practices around the

area. Of the total area, the largest quote goes to the *Fraxinus excelsior* with the frequency percentage exceeding 65% within the stand (Figure 1). In figure 1, dominant species are *Fraxinus excelsior* (1), *Acer velutinum* (2), *Carpinus betulus* (3), *Alnus glutinosa* (4), *Quercus* spp (5), *Acer cappadocicum*(6), *Tilia platyphyllos* (7), *Ulmus glabra* (8), *Diospyros lotus* (9) and *Fagus orientalis* (10).



**Figure 1.** Percent of tree species composition in the studied *Fraxinus* stand

All of the tree individuals of this species underwent examination. The diameter at breast and stump diameter was measured at the height of 130 cm and 30 cm above ground level, respectively using a tape meter with millimeter precision. Total height and canopy height, the height of trunks with branches and trunks without branches were also measured by means of the Sonto clinometer by decimeter precision. The collected data was imported into the Microsoft Excel software to calculate diameter at breast height, stump diameter, total height, canopy height and the height of trunk with and without branches. Considering the broken canopy or even the canopies in the process of drying out and other unusual data (up to five percent totally), we had to omit some of the collected data. The calculated parameters were imported into the SPSS software version 19 and the average and standard deviation were calculated. Pearson coefficient of correlation was used to examine the significance of the correlations between parameters and it was ascertained that there were significant correlations between some parameters. Different regression models were tested for their goodness of fit with regard to the distribution of data cloud. Prior to application of the regression models, data was examined for its normality of distribution. Because of the lack of an appropriate statistical correlation between diameter at breast height and the height of trunk with and without branches, descriptive statistics was employed. A regression model was fitted for the parameters of diameter at breast height and slenderness coefficient according to the distribution of data cloud. With the aim of assessing the performance of the regression models and select the most appropriate, Root Mean Square Error validation model (RMSE) and

correlation coefficient were used. For this, one fifth of the data was selected randomly, put aside and used after the selection of the model to examine the differences between observed and measured quantities. RMSE is a key parameter which is widely used for evaluation of the accuracy of spatial analysis and regression models in different studies (Siska and Hung, 2005). The criteria to determine the most appropriate regression model are the RMSE and  $R^2$ .

### 3. Results

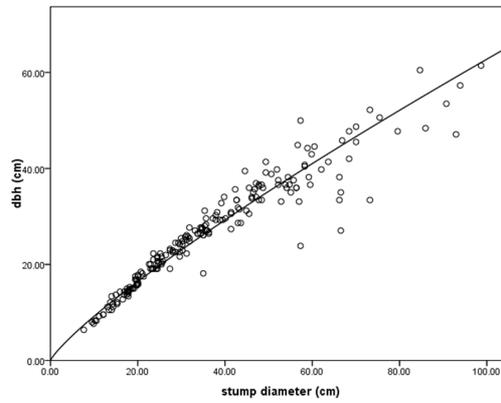
Results of the Pearson coefficient for correlation and analysis of significant differences are shown in Table 1.

**Table 1.** Pearson correlation coefficient among characteristics of *Fraxinus excelsior*

variables	sd	Dbh	toh	ch	htwob	htwib	trh
(sd)	1	0.949**	0.783**	0.753**	0.388**	0.316**	0.591**
(dbh)	0.949**	1	0.825**	0.772**	0.442**	0.321**	0.645**
(toh)	0.783**	0.825**	1	0.872**	0.586**	0.420**	0.851**
(ch)	0.753**	0.772**	0.872**	1	0.317**	0.260**	0.485**
(htwob)	0.388**	0.442**	0.586**	0.317**	1	0.292**	0.706**
(htwib)	0.316**	0.321**	0.420**	0.260**	0.292**	1	0.471**
(trh)	0.591**	0.645**	0.851**	0.485**	0.706**	0.471**	1

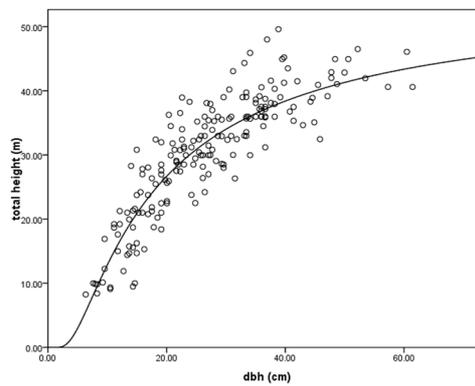
\*\*indicates significant correlation at 0.01 level. Abbreviations are stump diameter (sd), diameter at breast height (dbh), total height (toh), crown height (ch), height of trunk without branches (htwob), height of trunk with branches (htwib), and trunk height (trh).

Given the analysis performed on the data, regression models were obtained for the size-dependent characteristics. The criteria to choose the most appropriate regression models were the maximum adjusted determination coefficient ( $R^2$ ), minimum std. error of the estimate (MSE), maximum correlation coefficient ( $r$ ) and the minimum residual standard error (RMSE). We found an incremental power relation  $d = 1.356 \times d_{st}^{0.833}$  between stump diameter and diameter at breast height (Figure 2).



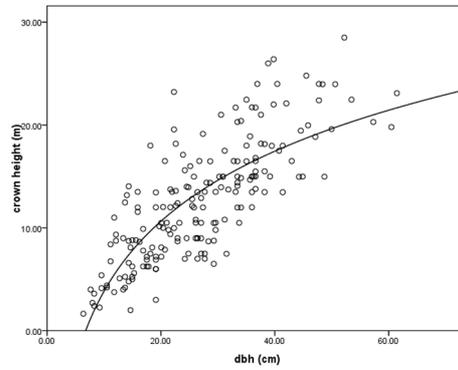
**Figure 2.** Relation between stump diameter and diameter at breast height

We could also establish the equation  $h_T = e^{(4.013 - (14.590/d))}$  between diameter at breast height and total height (Figure 3).



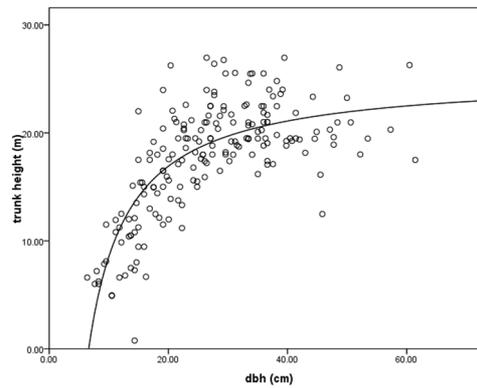
**Figure 3.** Relation between diameter at breast height and total height

We established the incremental logarithmic relation  $h_C = -18.789 + 9.826 \ln(d)$  between diameter at breast height and crown height (Figure 4).



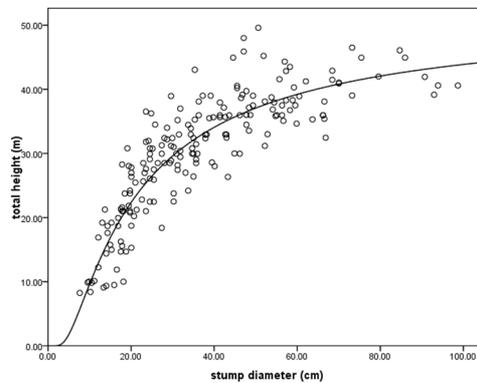
**Figure 4.** Relation between diameter at breast height and crown height

We also found the inverse relation  $h_t = 25.286 - (167.449/d)$  between diameter at breast height and trunk height of (Figure 5).



**Figure 5.** Relation between diameter at breast height and trunk height

Also, we established the sigmoid relation  $h_T = e^{(3.950 - (16.855/d_{st}))}$  between stump diameter and total height (Figure 6).



**Figure 6.** Relation between stump diameter and total height

To assess the relationship between diameter at breast height and the height of trunk without branches, diameter at breast height and the height of trunk with branches, descriptive statistics was employed. To this end, diameter at breast height was classified into the diameter classes of five centimeters and the height of trunk with and without branches was classified into height intervals of five meters. Presented in Table 2 and 3, are the results of this classification scheme.

**Table 2.** Distribution of tree frequency in categories of trunk heights with branch as related to the diameter categories

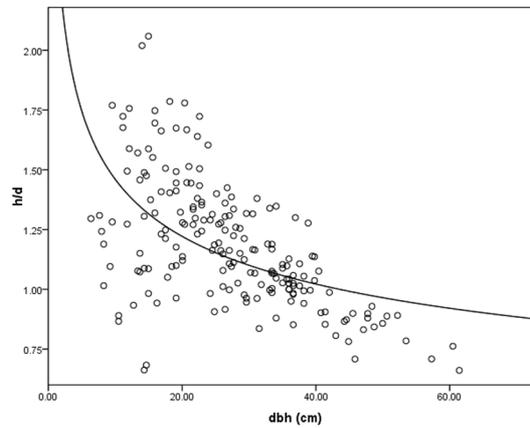
Dbh classes (cm)	Height of trunk with branch categories (m)				Total
	5	10	15	20	
10	11	3	0	0	14
15	27	8	1	0	36
20	18	16	3	0	37
25	19	12	5	0	36
30	15	8	5	0	28
35	25	11	1	0	37
40	7	4	3	1	15
45	2	1	2	0	5
50	3	3	3	0	9
55	0	1	1	0	2
60	0	0	2	0	2
Total	127	67	26	1	221

From table 2, one could see that the height of the trunk with branches mostly falls into the height category of 5 and 10 meters which match up with the diameter categories of 10 to 35 centimeters.

**Table 3.** Distribution of tree frequency in categories of trunk heights without branch as related to the diameter categories

Dbh classes (cm)	Height categories of trunk without branch (m)					Total
	5	10	15	20	25	
5	1	0	0	0	0	1
10	18	2	0	0	0	20
15	20	13	1	0	0	34
20	12	18	8	3	0	41
25	6	11	19	5	0	41
30	2	10	14	5	0	31
35	0	12	20	7	0	39
40	1	8	8	1	0	18
45	2	2	2	1	0	7
50	1	5	1	0	1	8
55	1	1	0	0	0	2
60	0	1	0	0	0	1
Total	64	83	73	22	1	243

As could be discerned in Table 3, the height of the trunks without branches mostly falls into the 5, 10 and 15 meters which again match up with the diameter categories of 10 to 35 centimeters. Slenderness coefficient and the diameter at breast height relationship is formulated as a decreasing equation  $H/D = 2.640 \times d^{-0.257}$  regarding other statistics with  $r=0.534$ ,  $R^2=0.282$  and  $SEE=0.187$  (Figure 7).



**Figure 7.** Relation between slenderness coefficient and the diameter at breast height

**Table 4.** Evaluation of various fitted models for *Fraxinus excelsior*

Model	RMSE	R <sup>2</sup>
Power (stump diameter and diameter at breast height)	3.38	0.94
Sigmoid (diameter at breast height and total height)	3.73	0.807
logarithmic (diameter at breast height and crown height)	3.96	0.602
inverse (diameter at breast height and trunkheight)	3.02	0.585
Sigmoid (stump diameter and total height)	4.09	0.803
Power (slenderness coefficient and the diameter at breast height)	0.17	0.285

**4. Discussion**

This study was taken place on the basis of mathematical-statistical analysis and descriptive interpretations for some of the characteristics of *Fraxinus excelsior*. Care must be taken, as the findings might not be applicable to other species and other locations. Results indicate that there is an increasing quadratic relationship between stump diameter of trees and the diameter at breast height for *Fraxinus excelsior* which could be applied for the estimation of the diameter at breast height for the cut trees. Namiranian (2004) found out that the relationship between the stump diameter and the diameter at breast height for *Fraxinus excelsior* is linearly correlated. In a study in Haftkhal forest of Sari, the relationship between age and changes in radial growth of trees was presented as a mathematical model (Amini et al., 2009).

The relationship between the diameter at breast height and the total height for *Fraxinus excelsior* was found to be sigmoidal. The final part of the diagram levels off after a period of incremental increase, and the established relationship could be hired to determine the total height for the cut trees. Namiranian (2004) demonstrated the Prodan relationship between the diameter at breast height and the total height for *Fraxinus excelsior*. In a study in Juniperus stand in Firoozkuh, the relationship

between total height and diameter at breast height was obtained as  $h = 1.5130498 + 1.1472926 \log(D)$  (Ramin et al., 2012).

There was a logarithmic relationship between the diameter at breast height and the total height which could be of great help to determine the height of the trees when lacking the information in the diameter at breast height. Namiranian (2004) found out that the relationship between the diameter at breast height and the total height of *Fraxinus excelsior* as increasing parabolic correlation. The established relationship for the diameter at breast height and the total height of the trunk was an inverse relationship. Moreover, the relationship between the stump diameter and total height was sigmoid. The important point left to be mentioned is that the studies of Namiranian (2004) were performed in a stand of uneven age structure. A preliminary comparison between Figure 4 (relationship between the diameter at breast height and total height of canopy) and Figure 5 (relationship between the diameter at breast height and total height of the trunk) leads to this conclusion that increase in canopy height as a result of increase in diameter at breast growth compared with total trunk height, will reach stability in longer run. The result reveals that as time passes and tree becomes older with greater diameter at breast height, canopy height will also expand and will account for the greater part of the height of the tree and will be drawn into the lower parts of the trunk.

From Table 2 it can be concluded that the height of the trunk with branches mostly falls into the height category of 5 and 10 meters which match up with the diameter categories of 10 to 35 centimeters. Besides, trees of greater diameters will have higher trunk elongation with fewer branches. Height distribution for trunks without branches which is presented in Table 3, speaks of the fact that diameter classes of 10 to 20 centimeters have trunk height (without branches) classes of about 5 meters, that diameter classes of 15 to 35 centimeters have trunk height (without branches) classes of about 10 meters, that diameter classes of 20 to 40 centimeters have trunk height (without branches) classes of about 15 meters.

## 5. Conclusion

We conclude that greater tree diameters will result in greater trunk (without branches) length. Thus, as tree diameter increases, this will clear larger parts of the main trunk from branches. Such tables could be used as the basis for planning forestry operations and posting marks. As our results indicate, increase in the diameter at breast height could affect slenderness coefficient and lower stands tree stability. The established relationship between these two variables follows a decreasing manner. In one study in pine stands of Alberta, Canada, researchers found that thinning caused increase in diameter at breast height and decrease in slenderness coefficient (Liu et al., 2003). These investigations are helpful in cultured parcels, and on this ground, application of such investigations is suggested for the areas under monitoring for developing protocols for culture of different species.

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