Swelling Bulk Density and its Application in Arya and Paris Model for Soil Moisture Curve Prediction

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Abstract

One of the Arya and Paris model (AP) drawbacks is setting saturated water content (θ_s) equal to total porosity ($f=1-\rho_b/\rho_p$), in which ρ_b and ρ_p are soil bulk and particle densities, respectively. For swelled soils with θ_s greater than f, using AP model leads to under prediction of water content at the measured suction values. The aim of this study was to introduce swelling bulk density ($\rho_{bs,}$: defined as the ratio of dry soil weight to its swelled volume) and its application in AP model to improve model efficiency for swelling soils. For this, we used 22 soil samples to check the accuracy of the model after improvement. Application of the ρ_{bs} improved the accuracy of the model compared to the conventional ρ_b. Employing ρ_{bs} instead of ρ_b increased the R^2 between measured and predicted water contents from 0.740 to 0.790 with a constant a 0.648 and 0.699 for variable a. Moreover, the intercept and slope of the regression line approached to 0 and 1, respectively.

Keywords: Arya and Paris model, swelling soil, swelling bulk density

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1. Introduction

Soil moisture curve (SMC), the functional relationship between soil matric potential (ψ) and soil moisture content (θ), may be one of the most important soil hydrologic properties. Measuring of θ at full ranges of ψ (0 – 1500 kPa) is tedious and time consuming. Several researchers have proposed different functions to express the relationship between ψ and θ (Brooks and Corry, 1964, Gupta and Larson, 1979, van Genuchten, 1980; Kosugi, 1994, etc.). One of the innovative indirect methods to predict SMC is Arya and Paris (AP) model (Arya and Paris, 1981 and Arya *et al.*, 1999) that uses PSD curve as the base of the model. Taking the saturated volumetric water content (θ_s) equal to the total porosity (f) is one of the drawbacks of the model that may occur in swelled soils (Arya and Paris, 1981). Because the model computes f using bulk (ρ_b) and particle (ρ_p) densities considering complete saturation. Regarding to swelling soils, however, θ_s may be greater than f leading to inaccurate or even erroneous results (Arya *et al.*, 1999; Mohammadi and Vanclooster, 2011 and Meskini-Vishkaee *et al.*, 2013). For this reason, the aim of the current research was to treat this drawback and to raise the accuracy of the AP model.

2. Material and methods
Correction for the swelling soil

Arya and Paris (1981) used the following equation to compute total porosity (f) and taking it as the saturated volumetric water content (θ_s).

$$(1) \quad f = 1 - \frac{\rho_b}{\rho_p} = \frac{\rho_p - \rho_b}{\rho_p}$$

The equation may be applicable to the non-swelling soils. Considering swelling soil cores, however, the phenomenon must be taken into the account. In this regard, initially it is needed to separate the swelling and non-swelling soils. The following ratio was used as a criterion to this purpose:

$$(2) \quad \frac{\theta_s}{f} > 1.1$$

It is assumed that (based on the authors observations) 10 percent of total difference between f and θ_s is due to measurement errors. So soils with $\theta_s/f > 1.1$ were considered as swelling soils. Considering swelling soils, authors suggested that swelling bulk density (ρ_{bs}) rather than conventional bulk density (ρ_b) should be applied to compute θ_s.

$$(3) \quad \theta_s = 1 - \frac{\rho_{bs}}{\rho_p} = \frac{\rho_p - \rho_{bs}}{\rho_p}$$

Where ρ_{bs} is called swelling bulk density and is defined as the following equation:

$$(4) \quad \rho_{bs} = \frac{m_{sd}}{V_{ts}}$$

Where m_{sd} is the weight of the oven dry soil and V_{ts} is the bulk volume of the swelled soil. Direct measurement of V_{ts}, however, can be time consuming and will need severe task, therefore we proposed the following approach to compute the ρ_{bs}.

$$(5) \quad \frac{\theta_s}{f} = \frac{\rho_p - \rho_{bs}}{\rho_p - \rho_b}$$

$$(6) \quad \rho_{bs} = \rho_b - \frac{\theta_s}{f}(\rho_p - \rho_b)$$

$$\text{if } \theta_s \approx f \text{ then } \rho_{bs} = \rho_b$$

$$\text{if } \theta_s > f \text{ then } \rho_{bs} < \rho_b$$

Where θ_s, ρ_p, and ρ_b are measured directly and f is computed value from Eq. 1.

Soil sampling and measurements

Twenty-two soil series with eight various textural classes (sand to silty clay loam) were selected from Karaj, Varamin and Urmia plains in Iran. Fifteen out of twenty-two selected soils/samples were suspicious to swelling.

Undisturbed core samples were taken by manual sampler from 0.05-0.1 m depth with three replications. Bulk and particle densities were measured according to Jacob and Clarke (2002) and soil texture using hydrometer (Gee, 2002). When samples were saturated, volumetric water content of each core sample at 2.5, 3.5, 7 kPa suctions were determined by water hanging column and at 10, 20, 30, 50, 100, 200, 300, 500, and 1000 kPa using pressure plate apparatus. Volumetric water contents in samples from Varamin and Urmia plains (10 out of 22) were measured up to 100 kPa suction.

Accuracy assessment

In order to compare the accuracy of the model, the computed ρ_{bs} from Eq. [6] and the measured conventional ρ_b were separately used to predict SMC using AP model (Arya and Paris, 1981). Criterions including root mean square error (RMSE) and determination coefficient (R^2) were employed to compare the model accuracy. The intercepts and slopes of regression line ($\theta_p = a + b\theta_m$) between the two sets of measured and predicted θ were also compared.

$$(7) \quad RMSE = \left[\frac{1}{n}\sum_{i=1}^{n}\left(X_{mi} - X_{pi}\right)^2\right]^{1/2}$$

Where $X_{m,I}$ and $X_{p,I}$ refer to the measured and predicted water contents (either using ρ_b or ρ_{bs}) at specified suctions, respectively.

3. Results and Discussion

Soil samples 3, 4, 7 and 11 to 22 showed considerable swelling (author's observation) during saturation stage (Table 1). The reported swelling criterion (θ_s/f), as depicted by Table 1, shows the occurrence of swelling in examined soils, as well. Figure 1 shows the drawback of Arya and Paris original model to predict the water content of swelling soils at various soil water potentials.

Table 1. Characteristics of the 22 examined soils for the investigation

No.	Soil texture	ρ_b (kg m^{-3})	ρ_p (kg m^{-3})	θ_s (m^{3}m^{-3})	f (m^{3}m^{-3})	$\dfrac{\theta_s}{f}$	ρ_{bs}ᵉ (kg m^{-3})
1	Sandy loam*	1460	2650	0.41	0.45	0.91	1554
2	Silty loam*	1390	2720	0.42	0.49	0.86	1572
3	Loam	1360	2600	0.59	0.48	1.23	1066
4	Loam	1390	2600	0.52	0.47	1.11	1237
5	Silty Clay Loam*	1270	2720	0.54	0.53	1.02	1251
6	Silty clay*	1230	2700	0.56	0.54	1.04	1180
7	Loam	1470	2560	0.55	0.43	1.28	1193
8	Sand*	1490	2590	0.43	0.42	1.02	1488
9	Clay loam*	1550	2500	0.40	0.38	1.05	1499
10	Sandy loam*	1520	2520	0.41	0.40	1.03	1484
11	Sandy clay loam	1620	2701	0.65	0.40	1.63	940
12	Sandy clay loam	1630	2689	0.75	0.39	1.92	672
13	Silty clay	1510	2563	0.73	0.41	1.78	681
14	Silty clay	1550	2490	0.76	0.38	2.00	608
15	Sandy clay loam	1390	2535	0.62	0.45	1.38	962
16	Loam	1280	2473	0.76	0.48	1.58	603
17	Silty loam	1440	2536	0.79	0.43	1.84	533
18	Loam	1540	2622	0.66	0.41	1.61	891
19	Loam	1480	2451	0.78	0.40	1.95	539
20	Silty loam	1500	2543	0.82	0.41	2.00	458
21	Silty clay	1460	2637	0.67	0.45	1.49	870
22	Loam	1460	2585	0.64	0.44	1.45	930
	mean	1454	2590	0.61	0.44	1.39	1036

* Non-swelling soils

θ_s : Measured water content at saturation

f: Total porosity computed by equation 1
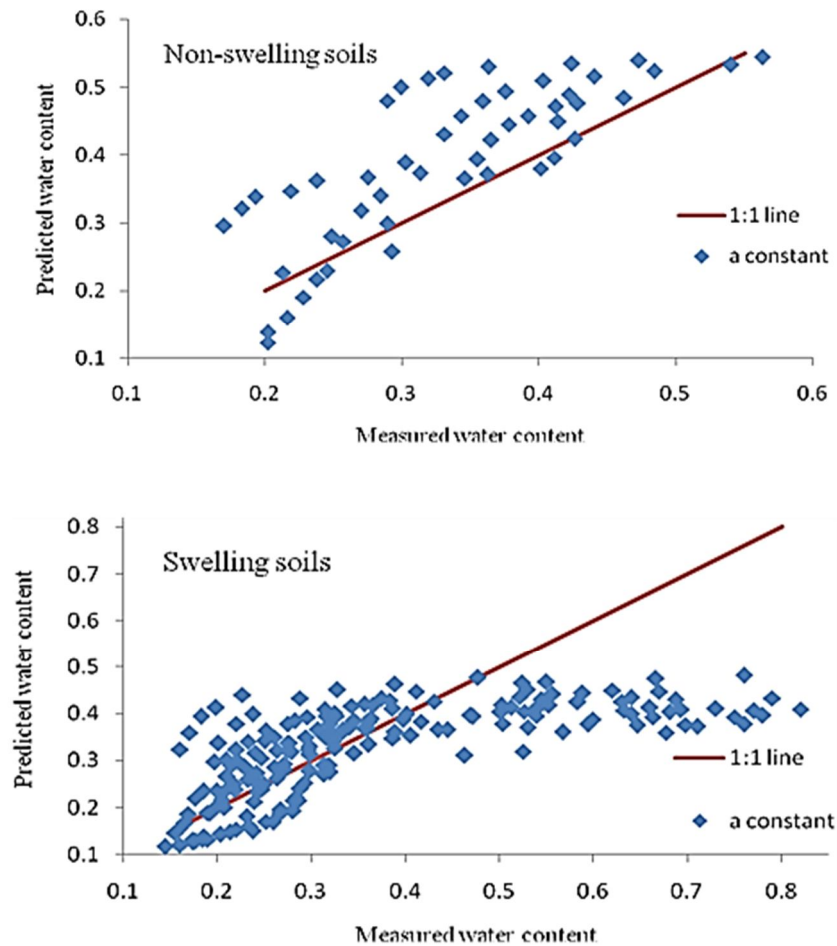
ᵉ: computed by equation 2

Figure 1. Relationship between predicted and measured θ by Arya and Paris (1981) original model for swelling and non-swelling soils

The RMSE and R^2 between measured and predicted θ by AP model (1981) using ρ_b and ρ_{bs} both for constant and variable a are reported in Table 2. Applying ρ_{bs} instead of ρ_b in swelling soils led to higher R^2 with 0.79 *vs.* 0.74 at constant and 0.699 *vs.* 0.648 at variable a.

The intercepts and slopes of the regression line between measured and predicted θ by AP model (1981) using ρ_b and ρ_{bs} both for constant and variable a are also reported in Table 3.

Table 3 shows that employing ρ_{bs} instead of ρ_b in swelling soils, although, led to slightly increase in RMSE with 0.148 *vs.* 0.124 for constant a and 0.206 *vs.* 0.134

for variable *a,* the mean intercept of the regression line approached closer to zero with -0.22 *vs.* -0.28 for constant *a* and -0.47 *vs.* -0.60 for variable *a*. The mean slope also became much closer to 1 with 1.16 *vs.* 1.92 for constant *a* and 1.51 *vs.* 2.59 for variable *a*. Generally, these results mentioned that using ρ_{bs} instead of ρ_b led to increase in R^2 and closer intercept to 0 and slope to 1. The ρ_b employment in AP model, especially for swelling soil with θ_s higher than *f*, leads to model failure by ranging θ between 0 and *f*. Contrary, ρ_{bs} employment automatically ranges θ between 0 and θ_s. Figure 2 shows the relationship between predicted (by using ρ_{bs} and ρ_b) and measured θ for swelling soils. As figure shows, the distribution of data around 1:1 line for the modified model emphasize the efficiency of the approach for swelling soils.

Table 2. The RMSE and R^2 between predicted and measured θ by using ρ_b and ρ_{bs} in Arya and Paris model (both for constant and variable *a*)

Soil No.	RMSE				R^2			
	ρ_b		ρ_{bs}		ρ_b		ρ_{bs}	
	*a*con.[¥]	*a* var.[¥]	*a* con.	*a* var.	*a* con.	*a* var.	*a* con.	*a* var.
1	0.049	0.099	0.033	0.075	0.820	0.692	0.807	0.682
2	0.055	0.076	0.017	0.026	0.805	0.778	0.800	0.779
3*	0.165	0.202	0.165	0.202	0.712	0.637	0.712	0.637
4*	0.044	0.072	0.074	0.113	0.616	0.532	0.627	0.544
5	0.093	0.118	0.099	0.124	0.789	0.731	0.790	0.731
6	0.156	0.171	0.171	0.187	0.708	0.660	0.710	0.661
7*	0.054	0.082	0.054	0.082	0.693	0.629	0.693	0.629
8	0.076	0.064	0.076	0.064	0.927	0.739	0.927	0.739
9	0.112	0.131	0.127	0.148	0.719	0.667	0.723	0.671
10	0.037	0.053	0.036	0.061	0.899	0.735	0.903	0.741
11*	0.126	0.106	0.027	0.087	0.967	0.861	0.981	0.923
12*	0.180	0.165	0.060	0.129	0.903	0.764	0.960	0.867
13*	0.140	0.147	0.239	0.284	0.658	0.587	0.722	0.641
14*	0.167	0.165	0.205	0.259	0.691	0.616	0.770	0.683
15*	0.082	0.064	0.030	0.087	0.955	0.854	0.973	0.895
16*	0.134	0.167	0.200	0.284	0.637	0.523	0.723	0.588
17*	0.152	0.168	0.260	0.329	0.634	0.549	0.715	0.617
18*	0.091	0.096	0.132	0.194	0.768	0.665	0.822	0.715
19*	0.151	0.160	0.225	0.298	0.655	0.556	0.754	0.641
20*	0.176	0.182	0.220	0.299	0.673	0.591	0.762	0.668
21*	0.112	0.130	0.217	0.258	0.733	0.660	0.774	0.699
22*	0.089	0.106	0.110	0.178	0.812	0.693	0.857	0.740
$\overline{X}^{\Gamma}$	0.124	0.134	0.148	0.206	0.740	0.648	0.790	0.699
$\overline{\overline{X}}^{\tau}$	0.111	0.124	0.126	0.171	0.762	0.669	0.796	0.704

*: swelling soils; ¥: constant and variable *a* in AP model (1981); Γ: mean of swelled soils; τ: total mean

Table 3. The intercept (a) and slope (b) of the regression $(\theta_p = a + b\theta_m)$ between predicted and measured θ by using ρ_b and ρ_{bs} in Arya and Paris model (both for constant and variable *a*)

Soil No.	Intercept (a) and slope (b)							
	ρ_b						ρ_{bs}	
	a con.		*a* var.		*a* can.		*a* var.	
	a	b	a	b	a	b	a	b
1	0.06	0.99	-0.13	1.09	0.05	0.77	-0.14	1.21
2	-0.04	0.98	-0.61	2.13	-0.08	1.23	-0.71	2.69
3*	-0.51	1.87	-1.19	3.27	-0.51	1.87	-1.19	3.27
4*	0.04	0.86	-0.24	1.43	0.05	0.74	-0.20	1.20
5	-0.51	1.87	-1.38	3.48	-0.50	1.83	-1.36	3.40
6	-1.57	3.75	-3.44	7.18	-1.53	3.56	-3.34	6.76
7*	0.01	0.86	-0.29	1.55	0.01	0.86	-0.29	1.55
8	0.13	0.64	0.06	0.69	0.13	0.64	0.06	0.69
9	-0.56	2.31	-1.31	4.29	-0.54	2.16	-1.28	3.98
10	0.09	0.73	0.01	0.84	0.09	0.70	0.01	0.80
11*	-0.04	1.64	-0.09	1.57	-0.03	1.06	-0.07	1.00
12*	-0.12	2.07	-0.18	2.00	-0.10	1.17	-0.15	1.10
13*	-0.74	3.16	-1.38	4.66	-0.58	1.58	-1.05	2.18
14*	-0.53	2.93	-1.03	4.20	-0.38	1.31	-0.73	1.73
15*	0.02	1.21	-0.04	1.22	0.02	0.90	-0.03	0.89
16*	-0.17	1.45	-0.37	1.77	-0.13	0.92	-0.28	1.04
17*	-0.49	2.40	-0.92	3.34	-0.36	1.20	-0.67	1.54
18*	-0.12	1.51	-0.34	2.00	-0.07	0.90	-0.25	1.13
19*	-0.34	2.23	-0.62	2.87	-0.25	1.07	-0.45	1.28
20*	-0.42	2.50	-0.85	3.48	-0.27	1.12	-0.56	1.42
21*	-0.67	2.66	-1.19	3.78	-0.58	1.66	-1.01	2.26
22*	-0.11	1.45	-0.24	1.67	-0.09	0.99	-0.20	1.09
$\bar{X}^{\Gamma}$	-0.28	1.92	-0.60	2.59	-0.22	1.16	-0.47	1.51
$\bar{X}^{\tau}$	-0.30	1.82	-0.72	2.66	-0.26	1.28	-0.63	1.92

*: swelling soils; ¥: constant and variable *a* in AP model (1981); Γ: mean of swelled soils; τ: total mean
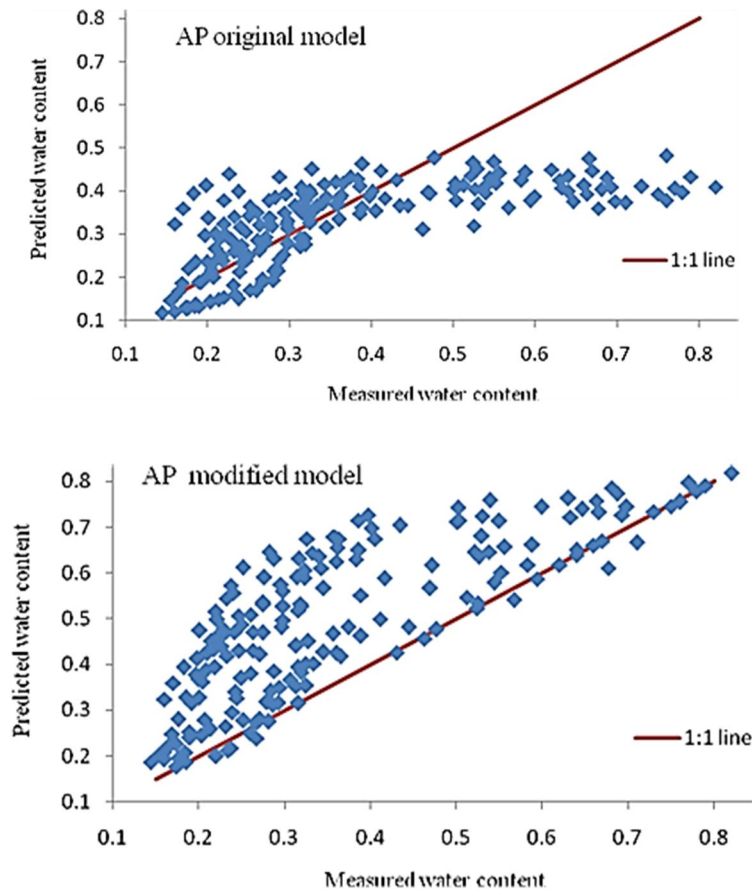
Figure 2. Relationship between predicted (by original and modified Arya and Paris (1981) (AP) model) and measured θ for swelling soils

4. Conclusion

Results showed that ρ_{bs} employment in AP model (1981) resulted in better application of the model for swelling soils. Although the R^2 increased slightly, the closer intercept to 0 and slope to 1 showed better use of the model for swelling soils.

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